MATH 1450 EXAM 3

NAME Key GRADE ——— -OUT OF 15 PTS

Answer each of following questions correctly for a full credit.

1. (3pts) **Optimization**

   (a) Find the point P on the line $x + y = 2$ that is closest to the point $A(-3, 1)$.

   (b) Find two positive numbers such that the sum of the first and twice the second is 320 and the product is a maximum.

(a) The distance function $D = (x+3)^2 + (y-1)^2$ is to be minimized. $D(x) = (x+3)^2 + (2-x-1)^2$ since $y = 2-x$

$$= x^2 + 6x + 9 + x^2 - 2x + 1$$

$$= 2x^2 + 4x + 10$$

Now $D'(x) = 4x + 4 = 0 \longrightarrow x = -1$ and thus

$y = 2 - (-1) = 3$ and $\boxed{P(-1,3)}$

(b) Let x and y be the numbers

$x + 2y = 320 \quad P = xy \longrightarrow P(y) = (320 - 2y)y$

$$= 320y - 2y^2$$

Hence $P'(y) = 320 - 4y = 0 \longrightarrow y = \frac{320}{4} = 80$ and $x = 160$

Thus, the numbers are $\boxed{80 \text{ and } 160}$

2. **Antiderivatives**

A-)(**4pts**) i) Evaluate $\int 3x^3 + 5\, dx$

ii) Evaluate $\int \frac{1+x}{x^2}\, dx$

iii) Evaluate $\int \cos(x)\, dx$

iv) Evaluate $\int \sec^2 t\, dt$.

(i) $\frac{3}{4}x^4 + 5x + c$

(ii) $= \int \frac{1}{x^2} + \frac{x}{x}\, dx = \int (x^{-2} + x^{-1})\, dx = -\frac{1}{x} + \ln|x| + c$

(iii) $= \sin x + c$ since $\frac{d}{dx}(\sin x) = \cos x$

(iv) $= \tan x + c$ since $\frac{d}{dx}(\tan x) = \sec^2 x$

B-) (**2pts**) A car traveling with velocity 24 m/s begins to slow down at time $t = 0$ with a constant deceleration of a = -6 m/s^2. Find:
(i) the velocity $v(t)$ at time t
(ii) the distance traveled before the car comes to a halt.

(i) $v(0) = 24$, $a(t) = -6 \rightarrow v(t) = \int a(t)\, dt = -6t + c$

at $t = 0 \rightarrow c = 24 \rightarrow v(t) = -6t + 24$

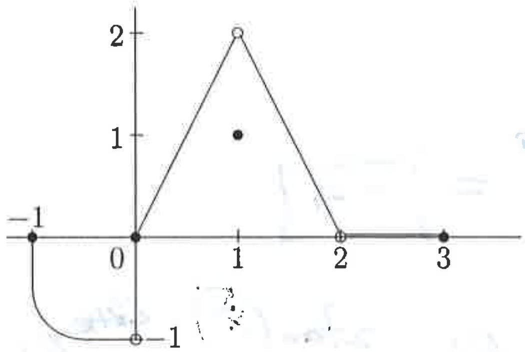
(ii) When the car stops, $v(t) = 0 \rightarrow -6t + 24 = 0 \rightarrow$ $t = 4s$

the position of the car $s(t) = \int v(t)\, dt = \int (-6t + 24)\, dt$
$= -3t^2 + 24t + c$

$s(0) = 0 \rightarrow c = 0 \rightarrow s(t) = -3t^2 + 24t$. So

at $t = 4 \rightarrow s(4) = -3(4)^2 + 24(4) = \boxed{48m}$

3. (**1.5pt**) Given graph of $f(x)$, answer the following question.



a) Find $f(1)$ $= 1$

b) What is $\lim_{x\to 1^-} f(x)$? $= 2$

c) What is $\lim_{x\to 1^+} f(x)$? $= 2$

d) What is $\lim_{x\to 1} f(x)$? $= 2$

e) Is f continuous at $x = 1$?, justify your response.

No since $\lim_{x\to 1} f(x) = 2 \neq 1 = f(1)$

4. (**1.5pt**) Find c to make the following function g a continuous function.

$$g(x) = \begin{cases} x^2 - c^2 & \text{if } x < 4 \\ cx + 20 & \text{if } x \geq 4 \end{cases}$$

$\lim_{x\to 4^\pm} g(x) = 4^2 - c^2 = 4c + 20 = g(4)$

So we have $+c^2 + 4c + 4 = 0$

$\Rightarrow (c+2)(c+2) = 0$

$\rightarrow \boxed{c = -2}$

5. **(3pts)** Evaluate the following limits. Justify your answer by showing your work!

(a) $\lim\limits_{x\to\infty} \dfrac{2+3x-4x^3}{x^3+1}$

$= \lim\limits_{x\to\infty} \dfrac{-4x^3}{x^3} = \boxed{-4}$

(b) $\lim\limits_{x\to\infty} 2x\tan(\frac{5}{x})$ $= \lim\limits_{x\to\infty} \dfrac{2\tan(\frac{5}{x})}{\frac{1}{x}} \overset{L'H}{=} \lim\limits_{x\to\infty} \left[\dfrac{-\frac{10}{x^2}\sec^2(\frac{5}{x})}{-\frac{1}{x^2}}\right]$

$= \lim\limits_{x\to\infty} 10\left[\dfrac{1}{\cos^2(\frac{5}{x})}\right] = 10\,\dfrac{1}{\cos(0)} = \boxed{10}$

(c) $\lim\limits_{x\to 1} \dfrac{x-1}{x^2-1}$

$= \lim\limits_{x\to 1}\left(\dfrac{(x-1)}{(x-1)(x+1)}\right) = \lim\limits_{x\to 1}\left[\dfrac{1}{x+1}\right] = \dfrac{1}{1+1} = \boxed{\dfrac{1}{2}}$

(d) $\lim\limits_{x\to 2} x^3-1$ $= [2]^3-1 = 8-1 = \boxed{7}$

(e) $\lim\limits_{x\to 0^-}(7+\frac{8}{x})$ $= \lim\limits_{x\to 0^-}\left(\dfrac{8}{x}\right) \longrightarrow \dfrac{8}{0^-} \longrightarrow \boxed{-\infty}$

(f) $\lim\limits_{x\to 5^+} \ln(x-5)$ $= \lim\limits_{x\to 5^+} \ln(0^+) \longrightarrow \boxed{-\infty}$